MEMORANDUM No.1

To: Members of seminar on Statistical Methodology

From: Howard Raiffa

Subject: An experiment involving objective and subjective uncertainties.

1. Do not discuss this memorandum with your colleagues before answering.

2. You are given two urns each containing <u>just</u> red and black balls. Urn No. 1 contains 50 red and 50 black balls. Urn No. 2 contains an <u>unknown</u> number of reds and an <u>unknown</u> number of blacks.

3. Suppose you are given two options:

<u>Option No. 1</u>: Select a color (R or B), announce it, and then take a single random drawing from urn No. 1.

<u>Option No. 2</u>: Select ~~a color (R or B)~~ the same color, announce it, and then take a single random drawing from urn No. 2.

Suppose, further, that no matter what option you choose the payoffs are as follows:

a) If <u>your</u> selection differs from <u>your</u> drawing you get nothing.

b) If <u>your</u> selection agrees with <u>your</u> drawing you gain $100.

Answer the following questions, keeping in mind <u>your</u> financial position as of <u>today</u>. Answers will be collected and kept anonymous.

<u>Question 1</u>: If you must choose either option No. 1 or option No. 2 which would you choose?

<u>Question 2</u>: If you were given the choice between taking no option or option No. 1 at a price, up to how much would you be willing to pay for option No. 1?

<u>Question 3</u>: If you were given the choice between taking no option or option No. 2 at a price, up to how much would you be willing to pay for option No. 2?

Options specify a given color for each urn: no chance for randomizing.

		R	B
5	0	10	0
0	5	5	5

a a b a b b

b a a b a b

$\frac{2}{3}$ $\frac{1}{3}$ $\frac{5}{3}+\frac{5}{3}$ a b a

$\frac{1}{2}\cdot 0+\frac{1}{2}\cdot 1$ 5 0 $\frac{2}{3}$

0 10 $\frac{1}{3}$ $\frac{20}{9}+\frac{10}{9}=\frac{30}{9}$ b a a

MEMORANDUM No. 2

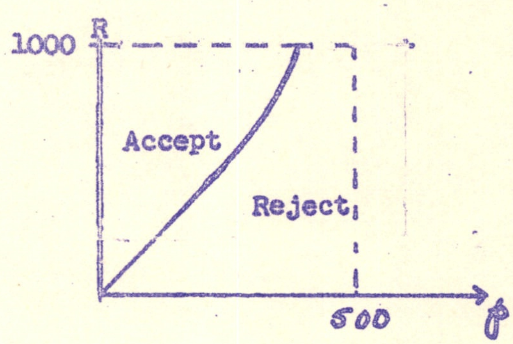
To: Members of seminar on Statistical Methodology

From: Howard Raiffa

Subject: An experiment involving attitudes toward money and gambling.

Prepare a set of complete and unambiguous instructions for your agent who will act in your behalf. Your agent will be given just one option which he can either accept or reject. However, you do not know the specific form of the option your agent will receive except that it will be of the following general form: It will involve a price p for the privilege of participating in a gamble with a 50-50 chance of receiving nothing or a reward of R dollars. Such an option will be denoted by (p, R). For example, if your agent is presented with option (27, 86) and your instructions tell him to accept, then: he pays $27; a fair coin is tossed; heads he gets $86 and tails he get nothing. You are further told R will not exceed $1,000.

Keeping your financial position, as of today, in mind, partition the (p, R) points into your own acceptance and rejection sets. For example, one possible partition might be of the form:



Memorandum No. 3 January 22, 1958

To: Members of Seminar on Statistical Methodology

From: Howard Raiffa

Subject: A continuation of memorandum No. 1.

We start out with the identical setup of memorandum No. 1. The new gimmick goes as follows:

Instead of a single trial there now will be a succession of ten trials. Each trial consists of the following four steps:

<u>Step 1:</u> Select a color (R or B) and announce it.

<u>Step 2:</u> Select an urn (no. 1 or 2) and draw a single random ball from the urn selected.

<u>Step 3:</u> You receive nothing if your announced color and the color of the withdrawn ball disagree; you receive $100 if they do agree.

<u>Step 4:</u> Return the withdrawn ball to its urn.

Formulate your <u>strategy</u> (in writing!) for the play of this ten-trial "game". Your set of instructions constitutes a bona fide strategy if and only if it satisfies this simple test: An agent acting in your behalf and using only your instructions must know <u>exactly</u> what to do at a given trial conditional upon his information about past trials. He should never have to return to you for further clarification of a situation not adequately dealt with in your instructions.

In this memo you're being asked what you would do if you had to play. In a later memo you will be asked: "Up to what amount would you be willing to pay for the privilege of playing this game?"

EX: Suppose they have to bet on the same color for 10 throws; and that Urn II contains 100 balls of same color.

EX: To urn II containing 50 R, add urn II containing 50 R or B balls.

or, Urn I has 49 R, 51 B; add 50 R or B balls.

EX: ~~Ex~~ $a \alpha b$; $a \beta b$

$a \alpha \underline{c}$? $a \beta c$

EX: R Y B How much will you pay for Y? for B? for Y ∪ B?

31 $\underbrace{\text{30 30}}_{60}$

for B after paying for Y?

MEMORANDUM No. 4

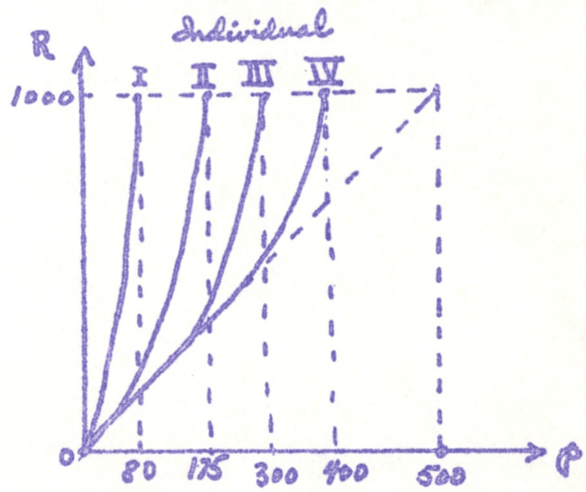
To: Members of Seminar on Statistical Methodology

From: Howard Raiffa

Subject: A Group Variation of Memorandum 2.

<u>You</u> are the consultant to a team of four individuals. The team has been given the task of giving a complete set of instructions to an agent, acting in the team's behalf, telling him what (p,R) options to accept and what to reject. Besides their external problem of joint acceptance or joint rejection they have an internal problem: How should they share the risks and profits accruing from an option that is jointly accepted? The internal and external problems are intrinsically intertwined and they wish <u>you</u> to draw up a reasonable contract for the internal behavior of the group and a set of joint instructions for their external behavior.

The individual's accept-reject strategies are given below:



The problem is a difficult one and there is no unique "answer". But nevertheless be prepared to discuss some of the guiding principles that you think might be reasonable.

MEMORANDUM No. 5

To: Members of the Seminar on Statistical Methodology
From: Howard Raiffa
Subject: Are your subjective probabilities good enough?

Consider the following two options:

Option 1: Twenty-five students of this class will be picked at random. If any two students have the same birthdate you will receive $100; otherwise you get nothing.

Option 2: An urn contains 1000 balls, x of them red and 1000-x black. A ball is drawn at random. If it is red you get $100; black you get nothing.

Question a): If you had to make an independent decision in the next few minutes, for what value of x would you be indifferent between the two options?

b): What could you do to help yourself give a more thoughtful answer?

MEMORANDUM No. 6

To: Members of the Seminar on Statistical Methodology

From: Howard Raiffa

Subject: The "Law of Averages"

1. A coin has been tossed ~~9~~883 times resulting in 451 tails and 432 heads. However, trials ~~9~~878 to ~~9~~883 gave all tails. If you decided to bet on the next toss, how would you call? Why?

2. The following experiment has been conducted with a subject who claims to have extra-sensory perception. In a gambling experiment the subject was given 100 pennies. At each trial the subject wagered one penny against the experimenter that he could guess the outcome of a toss of a "reportedly" true coin. At the end of 10,000 trials the subject had a cumulated total of 273 pennies and <u>was ahead</u> (i.e. had more than 100 pennies) <u>for 9,573 trials</u>! The experimenter was at first shattered by these results since he does not believe in extra-sensory perception. But later he realized that perhaps the coin is not really "true" since in the 10,000 trials only 4,875 heads appeared and perhaps this bias could have accounted for the astounding 9,573 figure. Comment.

3. What do you think of the following gambling system for betting against the house? Our chances of winning an even money wager at any trial is .45 (a little less than the "fair" .50). The house limit on a single wager is $1000; our initial capital is $5000. Our procedure is the following: We start a "round" of trials by writing down the three numbers

1 2 3.

In a given round we will add certain numbers to this list and delete other numbers in a manner to be prescribed below. At any trial we bet the sum of the first and last numbers on our list. For example at stage one we bet $1 + 3 = 4$. If we lose we add 4 to the list giving

1 2 3 4.

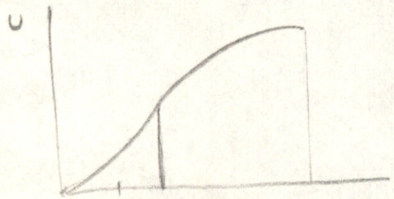
Next we would bet $1 + 4 = 5$. If we lose we add 5 to the list, giving

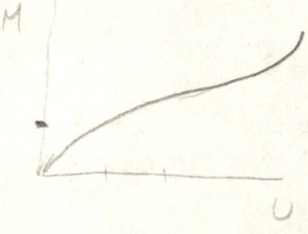
1 2 3 4 5.

Next, we would bet 1 + 5 = 6. Suppose now we win. Then we delete the first and last numbers on the list, giving

2 3 4.

We next bet 2 + 4 = 6. If we lose we add 6; if we win we delete 2 and 4. Each time we lose we add a single number to the list. Each time we win we delete two numbers from the list (except when there is only one number left). It canbbe shown that when all the numbers are deleted (i.e. a round is through) we have won 1 + 2 + 3 = 6 dollars. Sooner or later all the numbers will be deleted since we take two away .45 of the time in the long run and add only one .55 of the time in the long run.





a	a	b
b	a	a
a	b	a
a	a	b
b	a	a
a	b	a
b	a	a

0 X 10 6 0 Z 10 16

$p \cdot u(0) + (1-p)u(10) = u(6) \qquad p = \frac{u(6) - u(10)}{u(0) - u(10)}$

$p \cdot u(0) - p\, u(10) + u(10) = u(6)$

6 6

Memorandum No. 7

To: Members of the Seminar on Statistical Methodology

From: Howard Raiffa

Subject: What is "equally likely"?

The setting of this little parody is in a penitentiary. Three prisoners Able, Baker and Charlie and a Warden comprise our cast of characters. The parole board met yesterday in a secret meeting and decided to grant a parole to two inmates to be effective a week hence. It is common knowledge that only Able, Baker and Charlie were eligible for parole and it has leaked out that the parole board selected two prisoners by a random device giving each of the three prisoners an equal chance. As yet the board has not announced the lucky pair.

Able, our lead man, has been thinking hard about his chances and he's run smack into a philosophical paradox. Able originally figured out that the probability of his being paroled is 2/3. In particular, this followed since he considered the three events (A,B), (A,C), (B,C) -- [where (A,B) is elliptic for the event "Able and Baker are to be paroled", with a similar interpretation for (A,C) and (B,C)] -- to be <u>equally likely</u> and that 2 of these 3 mutually exclusive, mutually exhaustive <u>and equally likely</u> events would be favorable to him. Fine, so far, but now let us follow Able's present train of thought.

"I can't ask the warden if I'm one of the lucky ones. He won't tell me. But since we are such good friends I'm sure he will tell me in strict confidence the name of one of the parolee's other than myself. Perhaps with this information I can gain better insight into my chances. Suppose the warden says 'Baker is to be released', what then? Well then it's between Charlie and myself and I can't see why either of us has a better chance to be included. Thus conditional on the warden saying 'Baker', 'Able' and 'Charlie', are <u>equally likely</u>. Similarly, conditional on the warden saying 'Charlie', 'Able', and 'Baker' are <u>equally likely</u>. Well it seems that no matter what the warden says my chances have slipped from 2/3 to 1/2. Well, I won't ask him!"

<u>Question</u>: Does it seem to you the very fact that Able has even <u>thought about asking the warden his question</u> has caused his chances to go from <u>2/3 to 1/2</u>?

Pat Corazzi
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